

## RESEARCH-TO-PRACTICE SUMMARY

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### Acquisition and Measurement of Knowledge of Mathematical Development

Linda M. Platas  
*University of California, Berkeley*

This study provides preliminary evidence for the reliability and validity of the Knowledge of Mathematical Development Survey, which was designed to measure early childhood teachers' knowledge of early mathematical development. Findings also revealed two predictors of teachers' knowledge of mathematical development, classroom experience and completion of a course in mathematical development.

*Keywords:* Mathematical Development; Early Childhood Education; Professional Development; Preschool Curriculum; Preschool Teachers; Measurement

Mathematical proficiency is an important attribute throughout life and is currently an issue at the forefront of education policy development at all levels (National Research Council [NRC], 2009). This topic is especially relevant considering the advent of the Common Core State Standards (CCSS) in 45 states, which require greater conceptual understanding than many previous individual state standards. While the CCSS do not include standards at the preschool level, some states, like New York, have created aligned standards and more are sure to follow. Additionally, the predictive power of *early* math skills on later academic achievement (Duncan et al., 2007; Romano, Babchishin, Pagani, & Kohen, 2010) has elicited much interest in the *quality* of support of mathematical development in early childhood classrooms. Head Start's revised early learning framework emphasizes the predictive power of early math skills on later academic achievement in multiple domains (Administration for Children and Families, 2011, p. 16). Head Start and many state preschool guidelines now require teachers to provide mathematics instruction in the classroom, and call upon programs to implement curricula that ensure progress in this domain (Administration for Children and Families, 2011; Daily, Burkhauser & Halle, 2010).

Studies have revealed several challenges facing the early childhood education (ECE) field regarding the implementation of such policies: (a) current classroom support for early mathematical development is generally limited or non-existent (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; Varol et al., 2012); (b) teacher education and professional development programs frequently lack instruction in early mathematical development (Ginsburg, Lee & Boyd, 2008; Sarama, DiBiase, Clements & Spitler, 2004); and (c) the field lacks

research-based instruments that can reliably assess teachers' knowledge and evaluate the effectiveness of education and training in early mathematics (Maxwell, Field, & Clifford, 2006).

It has been suggested that a primary explanation for a lack of support for mathematical development in the early childhood classroom is that teachers rarely receive the preservice education necessary to support cognitive development, and that this particularly applies to early mathematical development (NRC, 2001, 2009). Inservice ECE professional development had traditionally focused on "developmentally appropriate" curriculum, literacy development, classroom management skills, and the use of play to promote socio-emotional development (Copley & Padron, 1998; Ginsburg et al., 2006; NAEYC, 2009). More recently, the field has focused on intentional teacher-child interactions, in particular in the domains of socio-emotional and early literacy development (Powell & Diamond, 2012; Sabol & Pianta, 2012). However, some promising inservice programs have emerged that target early math development and results indicate that the sustained professional development was successful not only in increasing the teachers' involvement in mathematical activities in the classroom, but also in increasing children's mathematical skills (Clements & Sarama, 2008; Platas, Klein, & Starkey, 2006).

Short-term workshops are a common method of providing training to teachers who are otherwise engaged in classroom teaching and have limited time resources. These workshops are generally offered over the course of just a few days (or even one day), however, unlike extended workshops, generally do not support sustainable change in knowledge or practice (Copple, 2004; Ginsburg et al., 2008). Further investigation into the contributions of workshops to teachers' knowledge of mathematical development would contribute to our understanding in the field of professional development.

## PURPOSE OF THE STUDY

The purpose of this study was to develop and examine the measurement properties of a new instrument for evaluating teachers' knowledge of mathematical development, the Knowledge of Mathematical Development (KMD) Survey. In addition, predictors including teaching experience, education level, exposure to professional development workshops in early mathematical development, and completion of a course in mathematical development were investigated for their contributions to teachers' knowledge of children's mathematical development.

## DEVELOPMENT OF THE KMD SURVEY

Over the past 30 years, researchers have been intensively studying young children's mathematical development and have come to understand a great deal about the developmental progression of children's mathematical understanding (Baroody, Lai, & Mix, 2006; Sarama & Clements, 2009). For example, can children count a row of items before they can count the same set size of items in a circle array? Or, does the arrangement of the items not matter? How does the size of the set of objects affect the strategies that children use to count? Is subtraction too advanced for preschoolers? What if the subtraction problem involves taking one cookie away from a set of three cookies? Children will be able to say that they have two cookies left. And, they'll definitely know they now have less than what they started with... all attributes of

understanding subtraction! Teachers must know the answers to these types of questions when creating effective and developmentally appropriate activities in the classroom.

Because the field of early mathematics is broad, the mathematical domains covered by this instrument are limited to number and operations. These domains are among the most researched topics in the field and, arguably, the most important in young children's mathematical development (Clements, 2004). The six subdomains of early number and operations included in this study are the verbal counting sequence, counting/numerosity, ordinal number words, addition and subtraction, division of sets ("fair-sharing"), and recognition and production of written number symbols.

The rationale for selecting items for the KMD Survey was based on three criteria: (a) supported by independent research and accepted as representative of the developmental progression by experts in the field; (b) common classroom activities; and (c) common activities found in preschool curriculum books. The development of the KMD Survey occurred over a multiphase study that consisted of four parts: item development, two pilot studies, and the study reported in this paper. In order to test the KMD Survey items for clarity and reliability, two pilot studies (20 and 53 participants, respectively) were conducted resulting in a 20-item instrument. The study presented in this paper utilized this refined instrument with a sample of 346 pre- and inservice teachers (examples of KMD Survey items may be found in the Appendix) and provided data to answer two research questions:

1. Are the validity and reliability estimates of the KMD Survey sufficient to support its use in research, program planning, and classrooms?
2. What are the contributing factors in teachers' acquisition of knowledge of early mathematical development? How do level of education, years of experience, professional development workshops, and math development courses influence this knowledge?

## SUMMARY OF RESEARCH METHODS

Participants included ECE students from four community colleges in the San Francisco Bay Area, three California State Universities, and three Masters' programs in two states (western and eastern United States). At the Masters' level, students currently completing a Mathematical Development course or who had taken a Mathematical Development course as a prerequisite were recruited. All three of the Mathematical Development courses surveyed emphasized theories of development and teaching, understanding of young children's mathematical thinking, and the development of activities based on those theories and that understanding. These courses were three-credit semester classes. For participants without exposure to a math development course, instruction on mathematical development ranged from none to limited textbook discussion and, at the most, six hours of classroom interaction in curriculum and child development courses.

Instructors were contacted via e-mail and provided with an explanation of the study and a request for the author to visit classrooms to recruit participants and administer surveys. In all cases, permission was granted. Return rate average was 97%. Completion of the surveys took approximately 15 minutes.

Reliability of the survey was examined through measures of internal consistency. Validity was examined through a literature review, interviews with experts in the field, a review of current child mathematics assessments, cognitive interviews and an analysis of variance, comparing the KMD Survey scores of three groups/cohorts by item and total score. Three dimensions of importance were considered in setting the parameters for each of these cohorts. These were ECE education, ECE teaching experience, and completion of a mathematical development course. These three cohorts are described in the following paragraph.

The first cohort was representative of students just beginning their careers in the ECE field. These beginning community college and California State University ECE students had no teaching experience, no ECE education and had not completed a mathematical development course. The second cohort was representative of many teachers currently in the field. This cohort included 3<sup>rd</sup> and 4<sup>th</sup> year university students with two or more years inservice experience and 12 or more units of ECE. Legislation passed in 2007 stated that by 2013, fifty percent of Head Start teachers must possess a Bachelor's or higher degree in ECE (Improving Head Start for School Readiness Act of 2007), making this cohort particularly representative of teachers in Head Start. The third cohort represents teachers in an M.A. program with two or more years of inservice experience, and enrollment in a mathematical development course. In all instances, participants in this cohort had either completed the entire course, or were nearing completion of the course (in both cases, study of number and operations had been completed). The third cohort represented teachers most likely to have the greatest amount of knowledge of children's mathematical development due to experience and education.

The third cohort was expected to perform significantly better than the second cohort, and both of these groups were expected to perform better than the first cohort. Since not all of the participants were assigned to a cohort (e.g., there were some students in Masters' programs who had less than two years of experience), the number of participants included in this analysis was reduced from the total participant pool.

While the above analyses provided information on the measurement properties of the KMD Survey, they do not provide information regarding the contribution of individual factors to knowledge of mathematical development. Other analyses, such as regression, can provide a much more detailed picture and can look at the effects of teaching experience, level of education, and completion of math development courses and professional development workshops on a teacher's knowledge of mathematical development. As noted earlier, education, experience and exposure to the study of mathematical development have been considered most effective in building teachers' knowledge of children's mathematical development. In order to measure the effects of these variables, a regression analysis was conducted on all complete cases.

## MAJOR FINDINGS

Estimates of reliability were obtained for the subset of participants who were included in the cohort analysis and for all participants who completed the KMD Survey. Cronbach's alpha results were fairly robust (.808 and .776, respectively). The analysis of variance showed that mean (average) KMD Survey scores differed significantly between cohorts, increasing as ECE education, experience and enrollment in a mathematical development course increased, providing support for validity of the KMD Survey instrument. As predicted, the third cohort performed significantly better than the second cohort, and in turn both of these groups performed

better than the first cohort (the means were 14.95, 12.00 and 10.30, respectively). While there was a significant difference between Cohorts 1 and 2 with an effect size of  $d = .48$ , the largest differences were those between Cohort 3 and the other cohorts, a difference of 4.65 points between Cohorts 1 and 3 ( $d = 1.51$ ), and 2.95 points between Cohorts 2 and 3 ( $d = 1.21$ ). Enrollment in a M.A. mathematical development course was associated with a significant increase in the participants' knowledge of early mathematical development as measured by the KMD Survey. These results, combined with evidence provided in the development phase and pilot studies, provided preliminary evidence in support of the reliability and validity of the KMD Survey.

With regard to the contributing factors in teachers' acquisition of knowledge of early mathematical development, the findings from the regression analysis indicate that when considering years of experience, level of education, mathematical development courses, and short-term workshops, only years of experience and completion of a math development course were related to how well teachers scored on the measure. For every year of experience, the total correct score on the KMD increased by .25 points and enrollment in a mathematical development course resulted in an increase of over two points.

## IMPLICATIONS FOR PRACTICE

It has frequently been argued that education makes a difference in the knowledge and practices of early childhood teachers (Bowman, 2011). As results from the regression analysis illustrated, it would take eight years of experience to equal the effect of the completion of a math development course. This outcome may explain why so many classrooms lack support for mathematical development, as the results suggest that it may take many years of experience to gain sufficient knowledge about mathematical development to provide support in the classroom. Results on the difference between exposure to a professional development math *workshop* and completion of a math development *course* suggest that sustained and deep interaction with the subject of math development is necessary for lasting results.

Major policy stakeholders in early education in the United States have issued statements concerning the urgency of including mathematics education in the early childhood education curriculum (NAEYC & NCTM, 2002; NRC, 2009). ECE teacher education programs must now begin to address the lack of education and training in this area of curriculum. The use of the KMD Survey could play a part in this enterprise, at multiple levels of the educational system: (a) individual instructors could use the survey to measure the knowledge of students at the beginning and end of a course to assess change in knowledge; (b) ECE programs could survey a cohort of students to inform the development of a mathematical development course; and (c) the use of the survey in multiple studies with differing interventions could provide useful comparisons across these interventions. The survey would also be useful in the development of any professional development program in which an increase in teachers' knowledge of children's mathematical development is a goal.

This study goes a considerable distance in ascertaining which factors contribute to teachers' knowledge of mathematical development. While the study will help to contribute to the literature on effective practices, more research is needed. A complete program of research on the quality and impact of ECE preparation in mathematics education would also include an examination of curricula, as well as investigations into the financial, programmatic and systemic

constraints on the implementation of effective mathematics education in the early childhood classroom.

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